Int. J. Heat Mass Transfer. Vol. 11, pp. 267–277. Pergamon Press 1968. Printed in Great Britain

TURBULENCE SCALE IN CHANNELS FORMED BY CLOSELY PACKED RODS

V. M. KASHCHEEV and E. V. NOMOFILOV
Energy-Physics Institute, Obnisk, U.S.S.R.

(*Received* 18 *November* 1966)

Аннотация—По методу Обухова А. М. получена формула для вычисления распределения масштаба турбулентности при турбулентном течении жидкости в каналах, образованных плотно упакованными круглыми стержнями при коридорном и шахматном расположении.

Приведены также функции, конформно отображающие поперечные сечения указанных каналов на внутренность единичного круга, необходимые для определения масштаба турбулентности.

NOMENCLATURE

a_{mk+1},	series coefficient;
C^{v}_{mk},	combination of mk-elements with respect to v;
$\Gamma(x)$,	gamma-function;
$F(\alpha, \beta, \gamma, x)$,	hypergeometric series;
h_{pq},	determinant elements;
h,	half the distance between parallel plates;
L,	circuit length;
$l(x, y)$,	turbulence scale;
$\tilde{l}$,	dimensionless turbulence scale;
m,	number of axially symmetric singly connected domain;
N,	internal normal;
R,	circle radius (R_e—exact, R_a—approximate);
r_0,	rod radius;
$w(\zeta)$,	function conformally representing area B to the inside of circle with radius R;
$w_1(\zeta)$,	function conformally representing area B to the inside of unit circle;
x, y,	Cartesian co-ordinates;
s,	arc;
ξ, η,	dimensionless Cartesian co-ordinates;
$\zeta = \xi + i\eta$,	dimensionless complex variable;
$\varkappa$,	constant multiplier;
ρ, φ,	polar co-ordinates;
ρ_0,	dimensionless rod radius;
ρ_G,	polar radius, corresponding to circuit points;
Re $w_1(\zeta)$, Im $w_1(\zeta)$,	real and imaginary parts of function $w_1(\zeta)$.

MANY problems on hydrodynamics and heat transfer are being solved by the methods of complex-variable functions and conformal representation of the regions. The work [1] deals with the general method on determination of turbulence scale in developed turbulent flow of liquid in straightline channels, whose cross-sections are arbitrary singly connected areas; the cross-section should be represented on a half-plane. The problems of hydrodynamics and heat transfer have been solved for the case of forced laminar convection in channels by mapping the cross-section shaped as an arbitrary singly connected area to the inside of a unit circle [2, 3] and a doubly connected area to the inside of a ring [4, 5].

The present paper is devoted to the determination of the turbulence scale in channels formed by closely packed rods either arranged in line or staggered. The knowledge of the scale of turbulence in complicated channels is necessary for obtaining a closed system of the equations of turbulent liquid flow.

In [1] it has been shown that the turbulence scale distribution over a cross-section of a closed straightline channel is given by the formula:

$$\tilde{l}(\xi, \eta) = \varkappa \frac{\operatorname{Im}[F(\zeta)]}{|F'(\zeta)|}. \tag{1}$$

Here the function $F(\zeta)$ conformally represents the channel cross-section, which is an arbitrary singly connected area, to a half plane. Expression (1) is obtained on the basis of the von Kármán principle on local similarity of turbulent processes at different flow points and assuming that:

(a) the turbulence scale distribution is determined only by geometrical properties of the channel and does not depend upon dynamic flow characteristics;

(b) the following condition is satisfied at the flow boundary:

$$\left(\frac{\partial l}{\partial N}\right)_G = \varkappa.$$

For the simplest cross-sections (circle, semi-circle, infinite band) turbulence scales have been calculated in [1]; the author has used the known functions conformally representing the above areas onto the half-plane.

In the available reference literature on conformal representation [6, 7] we could not find functions representing the transformations of the cross-sections of the channels considered in this paper (i.e. insides of right circular triangle and circular square) onto the half-plane.

To find the mapping functions, use the method of successive representation: the areas under consideration are first represented approximately (with any degree of accuracy) to the inside of the unit circle, and then the latter is precisely presented onto the half-plane. It should be noted that in [8] the following is obtained:

$$\zeta = f(w) = w \frac{\int_0^1 t^{-\frac{g+1}{2}} (1-t)^{\frac{g-1}{2}+\frac{1}{m}} (1 - w^m t)^{\frac{g-1}{2}-\frac{1}{m}} \mathrm{d}t}{\int_0^1 t^{-\frac{g+1}{2}} (1-t)^{\frac{g-1}{2}-\frac{1}{m}} (1 - w^m t)^{\frac{g-1}{2}+\frac{1}{m}} \mathrm{d}t} \tag{2}$$

which uniquely transforms the area of the circle $|w| < 1$ to the area B, which is the inside of the right circular m-angled figure with the centre at $\zeta = 0$, with one of the apexes at $\zeta = 1$ and internal angles πg, $0 \leqslant g \leqslant 2$, the mapping function obeying the conditions $f(0) = 0$, $f(1) = 1$.

Function (2) is easily reduced to the form:

$$\zeta = f(w) = \frac{\Gamma\left(\frac{1+g}{2}+\frac{1}{m}\right)\Gamma\left(1-\frac{1}{m}\right)}{\Gamma\left(\frac{1+g}{2}-\frac{1}{m}\right)\Gamma\left(1+\frac{1}{m}\right)} w \frac{F\left(\frac{1-g}{2}+\frac{1}{m}, \frac{1-g}{2}, 1+\frac{1}{m}, w^m\right)}{F\left(\frac{1-g}{2}-\frac{1}{m}, \frac{1-g}{2}, 1-\frac{1}{m}, w^m\right)}. \tag{3}$$

A particular case of expression (3) for $m = 3$ is given in [6]. Formula (3) might be used for obtaining functions conformally representing right circular m-angled figures and, in particular, our area B_1 ($m = 4$, $g = 0$) and B_2 ($m = 3$, $g = 0$) inscribed a unit circle, to the inside of the circle of radius R which is obviously equal to [see formula (3)]:

$$R_e = \frac{\Gamma\left(\frac{1+g}{2}+\frac{1}{m}\right)\Gamma\left(1-\frac{1}{m}\right)}{\Gamma\left(\frac{1+g}{2}-\frac{1}{m}\right)\Gamma\left(1+\frac{1}{m}\right)}. \tag{4}$$

However, the transformation of expression (3) presents a cumbersome problem. Therefore, these very functions have been found by the variational method [9].

If the singly connected area B, prescribed in the complex plane ζ and bounded by a piece-wise smooth contour G, is conformally transformed by the function $w = w(\zeta)$ into the circle with the origin of the co-ordinates as the centre, so that the point "a" from the area B which may be taken as the co-ordinate origin, transforms into the centre of the circle [$w(0) = 0$], and the given direction at the point $a = 0$ taken as the positive direction of the real axis coincides with the positive direction of the real axis of the transformed area [$w(0) = 1$], then expanding the function $f(\zeta) = \sqrt{[w(\zeta)]}$ into the Szegö polynomials which are orthogonal on the contour G and using the extremum property of the contour of minimum length in transforming the area B into the circle the following expression is obtained for the function $w = w(\zeta)$:

$$w(\zeta) = \frac{1}{k^2(0,0)} \int_0^{\zeta} k^2(0,\zeta)\, d\zeta \tag{5}$$

where

$$k(0,\zeta) = \sum_{n=0}^{\infty} \overline{P_n(0)}\, P_n(\zeta). \tag{6}$$

$P_n(\zeta)$ are the Szegö polynomials calculated by the formula:

$$P_n(\zeta) = \frac{1}{\sqrt{(D_{n-1} D_n)}} \begin{vmatrix} h_{00}, & h_{10}, & \ldots, h_{n0} \\ h_{01}, & h_{11}, & \ldots, h_{n1} \\ \ldots & \ldots & \ldots \\ h_{0,n-1}, & h_{1,n-1}, & \ldots, h_{n,n-1} \\ 1 & \zeta & \zeta^n \end{vmatrix} \tag{7}$$

$$D_0 = 1, D_n = \begin{vmatrix} h_{00}, & h_{10}, & \ldots, & h_{n0} \\ h_{01}, & h_{11}, & \ldots, & h_{n1} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ h_{0,n} & h_{1n}, & \ldots, & h_{nn} \end{vmatrix} \tag{8}$$

$$h_{pq} = \frac{1}{L}\int_G \zeta^p \bar{\zeta}^q \, ds, \qquad h_{pq} = \bar{h}_{qp} \tag{9}$$

$$L = \int_G ds. \tag{10}$$

As usual, multi-mode quantities are designated through bars. Function (5) represents the area B to the inside of the circle of radius R

$$R = \frac{L}{2\pi}\,\frac{1}{k(0,0)}. \tag{11}$$

To obtain the function representing the area on a unit circle, it is enough to divide formula (5) by formula (11), then

$$w_1(\zeta) = \frac{2\pi}{L}\,\frac{1}{k(0,0)}\int_0^{\zeta} k^2(0,\zeta)\,d\zeta. \tag{12}$$

The mapping functions for the areas formed by closely packed rods (areas B_1 and B_2, Fig. 1) were calculated by formulae (5–12).

The mapping functions are obtained in the form of an infinite series

$$w_1(\zeta) = \sum_{k=0}^{\infty} a_{mk+1}\zeta^{mk+1}. \tag{13}$$

It should be noted that for uniform convergence of series (13) in the area B it is enough to have the condition on the segmentary smoothness of the contour in the mapping area which is satisfied for the areas B_1 and B_2.

Terminating the series at some term, we obtain the approximate formula.

It is known that the function

$$F(w_1) = i\,\frac{1 + w_1}{1 - w_1} \tag{14}$$

represents the area of the unit circle $|w_1| < 1$ on the half plane. If formula (13) is substituted for w_1 in formula (14), then we obtain the function conformally representing the right circular m-angled figures on the plane:

$$F(\zeta) = i\,\frac{1 + \sum_{k=0}^{\infty} a_{mk+1}\zeta^{mk+1}}{1 - \sum_{k=0}^{\infty} a_{mk+1}\zeta^{mk+1}}. \tag{15}$$

On substituting formula (15) into formula (1), after necessary transformations, we find for the turbulence scale

$$\tilde{l}(\xi, \eta) = \varkappa \frac{1 - \{[\mathrm{Re}\, w_1(\zeta)]^2 + [\mathrm{Im}\, w_1(\zeta)]^2\}}{2\{[\mathrm{Re}\, w_1'(\zeta)]^2 + [\mathrm{Im}\, w_1'(\zeta)]^2\}^{\frac{1}{2}}} \tag{16}$$

where

$$\begin{aligned}
\mathrm{Re}\, w_1(\zeta) &= \sum_{k=0}^{\infty} a_{mk+1}\, \xi^{mk+1} \sum_{\nu=0,2,4\ldots}^{mk+1} (-1)^{\frac{\nu}{2}} C^{\nu}_{mk+1} \xi^{-\nu} \eta^{\nu} \\
\mathrm{Im}\, w_1(\zeta) &= \sum_{k=0}^{\infty} a_{mk+1}\, \xi^{mk+1} \sum_{\nu=1,3,5}^{mk+1} (-1)^{\frac{\nu-1}{2}} C^{\nu}_{mk+1} \xi^{-\nu} \eta^{\nu} \\
\mathrm{Re}\, w_1'(\zeta) &= \sum_{k=0}^{\infty} (mk+1)\, a_{mk+1}\, \xi^{mk} \sum_{\nu=0,2,4}^{mk} (-1)^{\frac{\nu}{2}} C^{\nu}_{mk} \xi^{-\nu} \eta^{\nu} \\
\mathrm{Im}\, w_1'(\zeta) &= \sum_{k=0}^{\infty} (mk+1)\, a_{mk+1}\, \xi^{mk} \sum_{\nu=1,3,5}^{mk} (-1)^{\frac{\nu-1}{2}} C^{\nu}_{mk} \xi^{-\nu} \eta^{\nu}.
\end{aligned} \tag{17}$$

In the cylindrical co-ordinate system, formulae (17) are of the form:

$$\begin{aligned}
\mathrm{Re}\, w_1(\zeta) &= \sum_{k=0}^{\infty} a_{mk+1}\, \rho^{mk+1} \cos(mk+1)\varphi \\
\mathrm{Im}\, w_1(\zeta) &= \sum_{k=0}^{\infty} a_{mk+1}\, \rho^{mk+1} \sin(mk+1)\varphi \\
\mathrm{Re}\, w_1'(\zeta) &= \sum_{k=0}^{\infty} (mk+1)\, a_{mk+1}\, \rho^{mk} \cos mk\varphi \\
\mathrm{Im}\, w_1'(\zeta) &= \sum_{k=0}^{\infty} (mk+1)\, a_{mk+1}\, \rho^{mk} \sin mk\varphi.
\end{aligned} \tag{18}$$

The prime denotes the derivative $w_1(\zeta)$ with respect to ζ. We now find the coefficients a_{mk+1} of series (13) for the areas B_1 and B_2.

In the co-ordinate system (Fig. 1a) the contour G_1 for the area B_1 is described by the formulae:

$$\begin{aligned}
\cup\, ab&: x^2 + (y - r_0\sqrt{2})^2 = r_0^2 \\
\cup\, bc&: (x - r_0\sqrt{2})^2 + y^2 = r_0^2 \\
\cup\, cd&: x^2 + (y + r_0\sqrt{2})^2 = r_0^2 \\
\cup\, da&: (x + r_0\sqrt{2})^2 + y^2 = r_0^2.
\end{aligned} \tag{19}$$

Passing to the dimensionless co-ordinates $\xi = x/r_0$ and $\eta = y/r_0$; in this case expressions (19) assume the form:

$$\begin{aligned}
\cup\, ab&: \xi^2 + (\eta - \sqrt{2})^2 = 1 \\
\cup\, bc&: (\xi - \sqrt{2})^2 + \eta^2 = 1 \\
\cup\, cd&: \xi^2 + (\eta + \sqrt{2})^2 = 1 \\
\cup\, da&: (\xi + \sqrt{2})^2 + \eta^2 = 1.
\end{aligned} \tag{20}$$

According to formula (9), taking into account (20), we determine the elements of the determinant (8)

$$h_{pq} = \begin{cases} 0, \; p - q \neq 4k, & k = 0, 1, 2 \ldots \\ \dfrac{4}{\pi}\displaystyle\int_0^1 [3 - 2\sqrt{(2 - \xi^2)}]^{\frac{p+q}{2}} \cos(p - q)\,\varphi_1 \frac{d\xi}{\sqrt{(2 - \xi^2)}} \\ p - q = 4k, & k = 0, 1, 2 \ldots \end{cases} \quad (21)$$

$$\varphi_1 = \arctan \frac{2 - \sqrt{(2 - \xi^2)}}{\xi}.$$

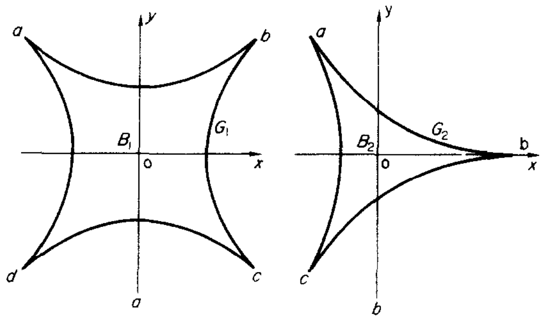

FIG. 1. Cross-section of the channels formed by closely packed rods arranged in line (a) and staggered (b).

It should be noted that integrals (9) with respect to contour (20) are taken analytically at fixed p and q. However, the unwieldiness of the expressions compels us to carry out numerical calculations of the integrals. Expressions (21) are therefore given in the form convenient for numerical integration.

Confining ourselves to the eighth-order Szegö polynomials, according to formula (5) we have the function

$$w(\zeta) = \zeta + 1{\cdot}733\zeta^5 + 3{\cdot}064\,\zeta^9 + 2{\cdot}704\,\zeta^{13} + 0{\cdot}9254\,\zeta^{17} \quad (22)$$

which represents the area B_1 inside the circle of radius R; the exact value of the circle radius calculated by formula (4) is equal to $R_e = 0{\cdot}4569$ while the approximate value of the circle radius given by formula (11) is $R_a = 0{\cdot}4693$. The degree of the proximity of function (22) lies not only in the finite number of the terms of the series but also in the fact that the series coefficients, starting from the second, are determined approximately. Therefore, to obtain the function conformally representing the area B_1 inside the unit circle $|w_1| < 1$, it is useful to divide expression (22) by the exact value of the radius R_e, i.e.

$$w_1(\zeta) = 2{\cdot}188\,\zeta + 3{\cdot}880\,\zeta^5 + 6{\cdot}706\,\zeta^9 + 5{\cdot}918\,\zeta^{13} + 2{\cdot}025\,\zeta^{17}. \quad (23)$$

The first coefficient will have the exact value and consequently, as it is seen from formula (23), the large central zone of the area B_1 will be mapped with high accuracy.

Furthermore, instead of formula (23) we shall consider the function

$$w_1(\zeta) = 2{\cdot}188\,\zeta - 3{\cdot}880\,\zeta^5 + 6{\cdot}706\,\zeta^9 - 5{\cdot}918\,\zeta^{13} + 2{\cdot}025\,\zeta^{17} \quad (24)$$

which represents the area B_1 turned at an angle equal to $\pi/4$, to the inside of the unit circle. This is necessary to obtain a unified formula for the calculations. The initial choice of the co-ordinate system (Fig. 1a) is dictated by the simplicity of the computing formulae for numerical integration.

The contour G_2 of the area B_2 (Fig. 1b) is described by the expressions:

$$\begin{aligned} \cup\, ab&: \left(x - \frac{r_0}{\sqrt{3}}\right)^2 + (y - r_0)^2 = r_0^2 \\ \cup\, bc&: \left(x - \frac{r_0}{\sqrt{3}}\right)^2 + (y + r_0)^2 = r_0^2 \\ \cup\, ca&: \left(x + \frac{2r_0}{\sqrt{3}}\right)^2 + y^2 = r_0^2 \end{aligned} \tag{25}$$

or in dimensionless variables

$$\xi = \frac{x\sqrt{3}}{r_0} \quad \text{and} \quad \eta = \frac{y\sqrt{3}}{r_0}$$

$$\begin{aligned} \cup\, ab&: (\xi - 1)^2 + (\eta - \sqrt{3})^2 = 3 \\ \cup\, bc&: (\xi - 1)^2 + (\eta + \sqrt{3})^2 = 3 \\ \cup\, ca&: (\xi + 2)^2 + \eta^2 = 3. \end{aligned} \tag{26}$$

Integrals (9) with respect to contour (26) for numerical integration are convenient to be written as:

$$h_{pq} = \begin{cases} 0, \; p - q \neq 3k, \qquad k = 0, 1, 2 \ldots \\ \dfrac{6}{\pi}\displaystyle\int_0^1 [7 - 2(\sqrt{3})\sqrt{(4 - \xi^2)}]^{\frac{p+q}{2}} \cos(p - q)\varphi_2 \frac{d\xi}{\sqrt{(4 - \xi^2)}};\; p - q = 3k, \qquad k = 0, 1, 2 \ldots \end{cases}$$

$$\varphi_2 = \arctan \frac{\xi}{\sqrt{(4 - \xi^2)} - 4/\sqrt{3}}. \tag{27}$$

The function

$$w(\zeta) = \zeta - 2{\cdot}612\,\zeta^4 + 6{\cdot}020\,\zeta^7 - 8{\cdot}377\,\zeta^{10} + 6{\cdot}715\,\zeta^{13} - 2{\cdot}853\,\zeta^{16} + 0{\cdot}4997\,\zeta^{19} \tag{28}$$

represents the area B_2 to the inside of the circle with the radius R; the Szegö polynomials including those up to the ninth order are used. Dividing function (28) by $R_e = 0{\cdot}3081$, we have

$$w_1(\zeta) = 3{\cdot}246\,\zeta - 8{\cdot}477\,\zeta^4 + 19{\cdot}54\,\zeta^7 - 27{\cdot}19\,\zeta^{10} + 21{\cdot}79\,\zeta^{13} - 9{\cdot}260\,\zeta^{16} + 1{\cdot}622\,\zeta^{19}. \tag{29}$$

Approximate functions (23) and (29) give the error of representation of linear dimensions ρ (for conformal representation the angles remain unchanged) which obviously increases from zero at $\zeta = 0$ to the maximum value (by absolute value) on the contour of the area under consideration. Fig. 2 gives the plot of the error of representation of the contours G_1 and G_2 on the circle of unit radius determined as

$$\varepsilon = |\, w_1(\rho_G, \varphi\,| - 1. \tag{30}$$

The subscript G indicates that the quantities are taken on the contour of the area.

Formula (16) is valid for the case when the function $w_1(\zeta)$ exactly represents the given area inside the unit circle. For approximate functions (24) and (29) it gives the overestimated results especially near the area contour at $\varepsilon < 0$ and even negative values at $\varepsilon > 0$; thus it is natural that the turbulence scale is not equal to zero on the contour of the cross-section of the channel, and condition (b) is not satisfied. According to formula (16) we obtain the exact value of turbulence scale at $\zeta = 0$

$$\frac{l(0, \varphi)}{r_0} = \frac{\varkappa}{2} \frac{R_e}{\rho_0}. \tag{31}$$

Since series (13) for the area contour especially in the vicinity of the points

$$\zeta = \rho_G \exp\left(i\frac{k\pi}{m}\right), k = 0, 1, 2 \ldots 2m - 1$$

converges rather slowly, the further increase in the number of its terms, as compared to formulae (24) and (29), is not effective because the inclusion of additional terms results in increased computation.

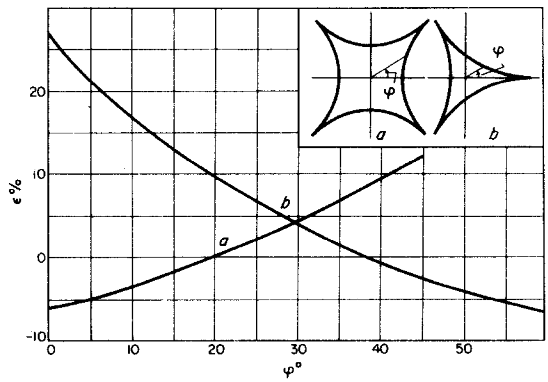

FIG. 2. The error ε of representation of contours G_1 (a) and G_2 (b) on the circle of unit radius.

To eliminate the error in determining the turbulence scale according to formula (16), we give the approximate formula

$$\tilde{l}(\rho, \varphi) = \frac{\kappa}{2} \frac{[1 + \varepsilon(\rho/\rho_G)^\tau]^2 - \{[\mathrm{Re}\, w_1(\zeta)]^2 + [\mathrm{Im}\, w_1(\zeta)]^2\}}{\{[\mathrm{Re}\, w_1'(\zeta)]^2 + [\mathrm{Im}\, w_1'(\zeta)]^2\}^{\frac{1}{2}}} \tag{32}$$

in which instead of writing unity for the square of the unit radius [in the numerator of formula (16)] we write down $R^2(\rho, \varphi)$ expressing the variable radius as

$$R(\rho, \varphi) = 1 + \varepsilon(\rho/\rho_G)^\tau. \tag{33}$$

The second term of formula (33) allows approximately for the increase in the error of representation of linear dimensions ρ from zero at $\zeta = 0$ to ε on the area contour. Among numerous similar expressions the power law relation is attractive because of its simplicity and gives quite satisfactory results as was shown by the subsequent calculations.

At $\varepsilon = 0$ formula (32) converts into formula (16), gives the value (31) at $\zeta = 0$ and is zero on the contour.

Imposing condition (b) on formula (32), to determine the exponent τ, we find:

$$\tau = \left| \frac{\rho_G}{\varepsilon(\varepsilon + 1)} \left(\frac{\{[\mathrm{Re}\, w_1'(\zeta)]^2 + [\mathrm{Im}\, w_1'(\zeta)]^2\}^{\frac{1}{2}}}{(1/\rho_G)\, \partial\rho_G/\partial\varphi \cos(\varphi, N) - \cos(\rho, N)} + [\mathrm{Re}\, w_1(\zeta)] \frac{\partial}{\partial\rho} [\mathrm{Re}\, w_1(\zeta)] + [\mathrm{Im}\, w_1(\zeta)] \frac{\partial}{\partial\rho} [\mathrm{Im}\, w_1(\zeta)] \right) \right| \quad \text{at} \quad \zeta = \zeta_G. \tag{34}$$

A comparison of the calculation results according to formulae (16) and (32) has been carried out for a channel formed by two infinite parallel plates being at a distance of $2h$ (infinite band). It is known [1] that in such a channel turbulence scale is calculated by the formula:

$$\frac{l}{h} = \frac{2\varkappa}{\pi} \cos\frac{\pi}{2}\frac{y}{h}. \tag{35}$$

Since the function

$$w_1(\zeta) = \tanh\frac{\pi}{4}\zeta \tag{36}$$

conformally represents this infinite band to the inside of the circle [7], then having expanded (36) into series

$$w_1(\zeta) = \frac{\pi}{4}\zeta - \frac{1}{3}\left(\frac{\pi}{4}\zeta\right)^3 + \frac{2}{15}\left(\frac{\pi}{4}\zeta\right)^5 - \dots \tag{37}$$

to calculate turbulence scale it is possible to use either formula (16) or formula (32). The calculation results obtained by means of formula (35), (32) and (16) are presented in Fig. 3.

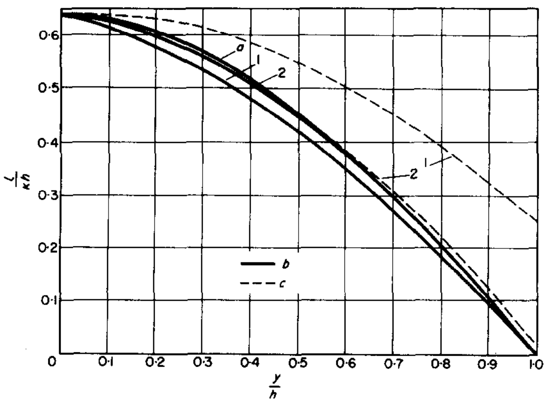

FIG. 3. Plot of the turbulence scale $l/\kappa h$ in a channel between two infinite parallel plates versus a dimensionless distance y/h.
(a) formula (35); (b) formula (32); (c) formula (16). (1) one term of expansion (37); (2) three terms of expansion (37)

The comparison of the results shows that when using the first term of expansion (37) to calculate l, formula (16) gives a large error while the calculation using formula (32) practically coincides with the exact value of turbulence scale.

The coefficients in series (24) and (29) were used for calculating the turbulence scale in channels formed by closely packed rods with the help of formula (32). Figure 4 shows the generalized dependence of the calculation of l/r_0 upon a relative distance ρ/ρ_G. The radius ρ_G corresponding to the points of the area contour is determined by the formula

$$\rho_G = \cos\varphi + \rho_0 \sin\varphi - \sqrt{[(\cos\varphi + \rho_0 \sin\varphi)^2 - 1]}\,;\ 0 \leqslant \varphi \leqslant \frac{2\pi}{m} \tag{38}$$

The insignificant scatter of the curves allows the choice of a very simple formula convenient for practical applications

$$\frac{l}{r_0} = \frac{\varkappa R_e}{2\rho_0} \cos\frac{\pi}{2}\frac{\rho}{\rho_G} \tag{39}$$

which over the angle range $\pi/36 < \varphi \leqslant \pi/m$ approximates the calculation results with an inaccuracy, not exceeding 10 per cent (Fig. 4). The approximate character of formula (32) as well as

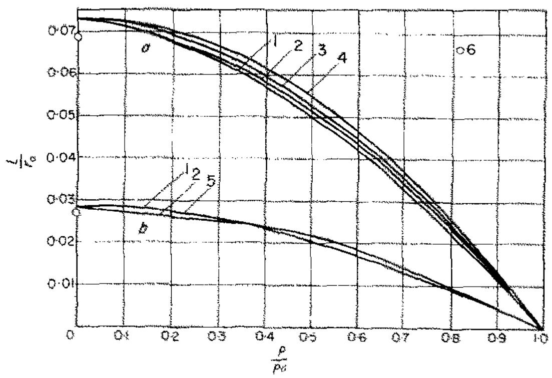

FIG. 4. The generalized dependence of the turbulence scale in channels formed by closely packed rods arranged in line (a) and staggered (b) on a dimensionless distance ρ/ρ_G for $\kappa = 0{\cdot}32$.

1: formula (39); 2: $\varphi = 15°$; 3: $\varphi = 30°$; 4: $\varphi = 45°$; 5: $\varphi = 60°$, formula (32); 6: data from [10].

the maximum error of mapping of angular zones of channels leads to the fact that in these zones the calculation results have noticeable deviations from the generalized curve. However, there are no physical restrictions to the application of formula (39) to the whole channel. In Fig. 4 are also given the values of the turbulence scales at the centre of the channels under consideration [10]. The divergence is not more than 5 per cent.

ACKNOWLEDGEMENTS

We wish to thank S. I. Morozova and T. M. Pushkareva for their help in carrying out calculations and preparing the paper.

REFERENCES

1. A. M. OBUKHOV, On the turbulence scale distribution in arbitrary cross-section flows, *Prikl. Mat. Mekh.* **6**, vyp. 2/3, 209–220 (1942).
2. U. A. SASTRY, Heat transfer by laminar forced convection in a pipe of curvilinear polygonal section, *J. Sci. Engng Res.* 7 (2), 281–292 (1963).

3. U. A. SASTRY, Solution of the heat transfer of laminar forced convection in non-circular pipes, *Appl. Scient. Res.* **A13** (4/5), 269–280 (1964).
4. U. A. SASTRY, Heat transfer by laminar forced convection in multiple connected cross-section, *Indian J. Pure Appl. Phys.* 3 (4), 113–116 (1965).
5. U. A. SASTRY, Viscous flow through tubes of doubly connected regions, *Indian J. Pure Appl. Phys.* 3 (7), 230–232 (1965).
6. H. KOBER, *Dictionary of Conformal Representation*. Dover, New York (1952).
7. W. VON KOPPENFELS and F. STALLMANN, *Praxis der Konformen Abbildung*. Springer, Berlin (1959).
8. G. M. GOLUZIN, *Geometric Theory of Complex-variable Functions*, pp. 105–110. Gos. Izd. Tehk.-Teor. Literatury, Moscow–Leningrad (1952).
9. L. V. KANTOROVICH and V. I. KRYLOV, *Approximate Methods of Higher Analysis*, pp. 400–406. Izd. Fizmat, Moscow–Leningrad (1962).
10. N. I. BULEEV, K. N. POLOSUKHIN and V. K. PYSHIN, Hydraulic resistance and heat transfer in turbulent fluid flow in lattice of rods, *Teplofiz. Vysok. Temp.* **2** (5), 749–756 (1964).

Abstract—According to Obukhov's method [1], the formula was obtained to calculate turbulence scale distribution in channels formed by closely packed rods arranged in-line or staggered.

The functions conformally representing the cross-sections of the above channels to the inside of the unit circle, which are necessary for determining turbulence scale, are also presented.

Résumé—En accord avec la méthode d'Obukhov [1], on a obtenu une formule pour calculer la distribution d'échelle de turbulence dans des canaux formés par des barres empilées régulièrement et alignées ou décalées.

Les fonctions permettant la représentation conforme des sections droites de ces canaux sur l'intérieur d'un cercle unitaire et qui sont nécessaires pour déterminer l'échelle de turbulence, sont également présentées.

Zusammenfassung—Nach der Methode von Obukhov [1] wurde die Gleichung für die Berechnung der Turbulenzgradverteilung in Kanälen aus fluchtend oder versetzt angeordneten Stabbündeln erhalten.

Die Funktionen der konformen Abbildung der Querschnitte obiger Kanäle auf das Innere des Einheitskreises, wie sie zur Bestimmung des Turbulenzgrades notwendig sind, werden ebenfalls angegeben.